Temporal ERGMs

A simple extension

Temporal ERGMs

- ERGMs are a general statistical framework for modeling cross-sectional network structure
  - A single ERGM predicts the *presence or absence* of a tie (and the static *prevalence* pattern of ties in the whole network)
  - Not the two dynamic processes of *tie formation* and *dissolution*

- To model network dynamics, we need temporal ERGMs (TERGMs)

- The TERGMs we are interested in can be represented by 2 ERGMs
  - One for the tie formation process
  - The other for the tie dissolution (or persistence) process

The relationship between ERGMs & TERGMs

- Intuition: If tie density is low in the cross-section, is that because
  - Ties form infrequently? (the *tie incidence rate*)
  - Ties form frequently, but don't last long? (tie *duration*)

- Recall the classic approximation formula from epidemiology
  - The relationship between disease *prevalence, incidence* and *duration*
  - Applies in this context also:

Prevalence ≈ Incidence x Duration

Tie density	Formation rate	Inverse of dissolution rate

TERGMs: Core idea

- Y (our network) is now indexed by time
- At each time step:
  - Evolution from Y_t to Y_{t+1} is a product of two processes:
    - Tie formation (on the empty dyads) and
    - Tie dissolution/persistence (on the tied dyads)
  - Each process is a draw from an ERGM
    - Two formulas: a formation formula and a dissolution/persistence formula
    - Two corresponding sets of statistics

TERGMs

ERGM: Conditional log-odds of a tie *existing*

$$logit\left(P\left(Y_{ij}=1\middle|\text{rest of the graph}\right)\right)=\boldsymbol{\theta}'\boldsymbol{\partial}\left(\boldsymbol{g}(\boldsymbol{y})\right)$$

TERGM:

Conditional log-odds of a tie *forming:*

$$logit\left(P\left(Y_{ij,t+1}=1\middle|\,Y_{ij,t}=0,\text{rest of the graph}\right)\right)=\boldsymbol{\theta}^{+\prime}\boldsymbol{\partial}\left(\boldsymbol{g}^{+}(\boldsymbol{y})\right)$$

Conditional log-odds of a tie *persisting:*

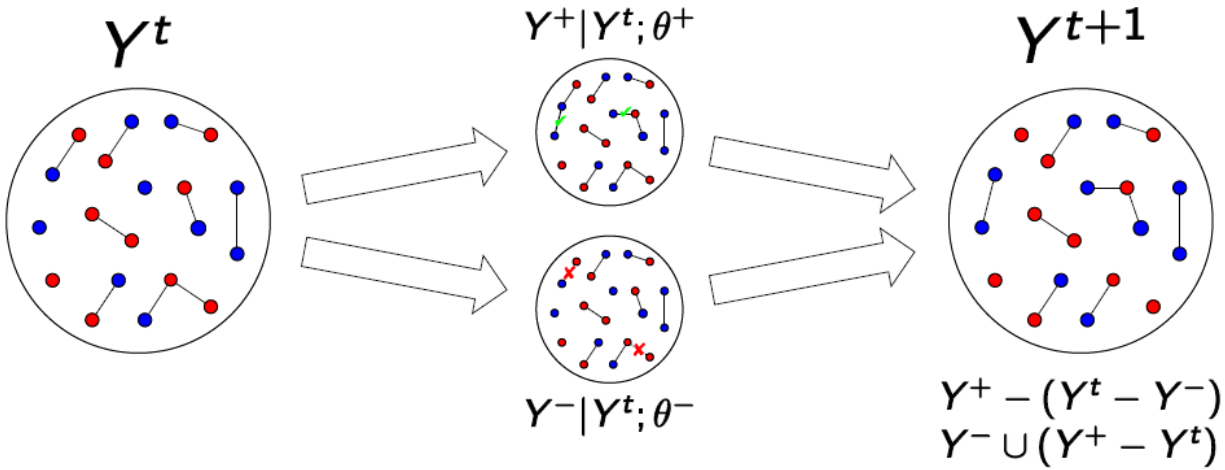
$$logit\left(P\left(Y_{ij,t+1}=1\middle|\,Y_{ij,t}=1,\text{rest of the graph}\right)\right)=\boldsymbol{\theta}^{-\prime}\boldsymbol{\partial}\left(\boldsymbol{g}^{-}(\boldsymbol{y})\right)$$

where:

$\boldsymbol{g}^{+}(\boldsymbol{y})$	= vector of network statistics in the formation model
$\boldsymbol{\theta}^{+}$	= vector of parameters in the formation model
$\boldsymbol{g}^{-}(\boldsymbol{y})$	= vector of network statistics in the dissolution model
$\boldsymbol{\theta}^{-}$	= vector of parameters in the dissolution model

These are separable TERGMs

- The two processes occur separately within a time step:



- Y^+ = network in the formation process after evolution
- Y^- = network in the dissolution process after evolution

Why separate formation from dissolution?

- Intuition: The determinants of tie formation are often different from those that influence dissolution.

- Interpretation: Because of this, we want model parameters that can be interpreted in terms of ties formed and ties dissolved. (We also need data that can allow us to estimate these).

- Estimation and Simulation: Our data typically take the form of a single cross-sectional network with information on relational durations (prevalence and duration), so this approach allows us to estimate incidence (formation) and match the observed data in simulation

TERGMs: Example of interpretation

Term = `~edges`

	θ is $+$	θ is $-$
Formation model	>50% of empty dyads have ties created during each timestep	<50% of empty dyads have ties created during each timestep
Dissolution (persistence) model	>50% of existing ties pre-served (fewer dissolved); longer average duration	<50% of existing ties pre-served (more dissolved); shorter average duration

Assuming time step is 1 day, what combo do you think is most common in long term partnership networks?

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TERGMs: Example of interpretation

Term = `~concurrent` (# of nodes with degree 2+)

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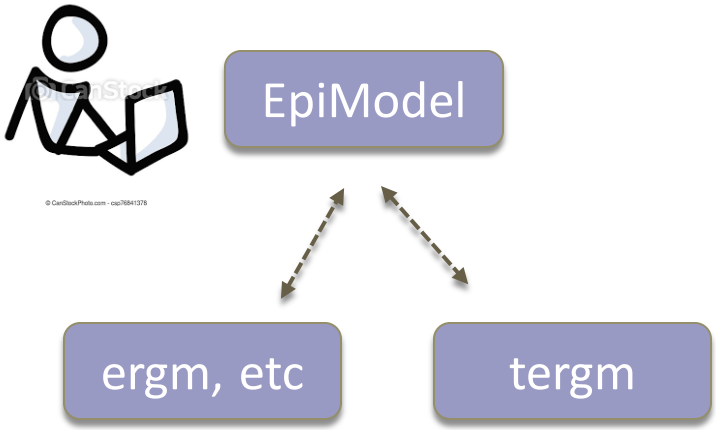
Why 2, and not 2+ in the interpretation of dissolution ?

No lab on tergms now, but …

- The Statnet package for temporal network models is called *tergm*
  - These temporal modeling tools are not implemented in `statnetWeb`
  - But TERGMs are the network model used in `EpiModel`
  - So you will learn to work with them via `EpiModel`, starting in module 4



Epimodel passes your code to the Statnet packages as needed, and gives you access to lots of additional functionality required for modeling epidemic and demographic processes

You can run these and other Statnet packages from R directly for standalone network analyses. See the online training materials at the statnet.org website for more information